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Thermal RC Ladder Networks

Packaging Technology Development

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APPLICATION NOTE

INTRODUCTION

This monograph discusses the nature of and differences between grounded−capacitor and non−grounded capacitor thermal RC−network models. It also explains how they may be used in describing and simulating thermal transient behavior, specifically of semiconductor device packages (though the principles are completely general).

Grounded−Capacitor Network

Figure 1 illustrates a typical “grounded−capacitor” thermal ladder network, also known as a Cauer network. In fact, any network topology of resistors might be chosen to represent a physical thermal system (i.e. not just a linear string of resistors, but just as well a star, a bridge, or whatever). The main advantage of a grounded−capacitor network is that it derives from the fundamental heat−transfer physics. Every “node” in the network is connected to thermal “ground” through a capacitor. It is simply convenient to draw the network as shown in Figure 1 because it resembles a ladder, though because the lower edge of each rung attaches directly to ground, the connections between the rungs are essentially through the resistors.

Because this network derives from the real physics (of which a more detailed discussion can be found in the following section on “non−grounded−capacitor” networks), there is at least a chance that experimental data from various points within the physical system can be correlated with specific individual nodes of the network model. As we move from junction to ambient, for instance, we might find physical locations correlating with the nodes in this order: silicon junction, back of silicon chip, edge of leadframe, lead (at package boundary), lead (at board interface), board (at some distance from package), and finally ambient. Of course, we may not have any intermediate location data with which to correlate, or intermediate data which we have might not happen to land “on” a node of the model (rather, somewhere in between nodes). Also, the physical system might not be well represented by such a simple chain of resistors, so no correlation might be possible except at the junction itself. (This is actually more typical than you might think, for in many environments, the heat flow follows at least two separate and distinct paths from junction to ambient, e.g. upward through the case, outward through the leads into the board, and downward through the air gap and thus directly to ambient on the back side of the board. When the heat flow is believed or known to flow along multiple parallel paths, it clearly would be better to model the system with a more complex network.) Only in the case where a single path to ground dominates heavily would such a simple linear resistor topology be expected to yield good correlations at the intermediate nodes. Nevertheless, the point is that there *could* be such a correlation.

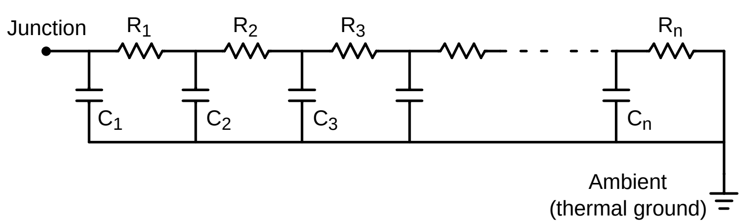

Figure 1. Grounded Capacitor Thermal Network (“Cauer” Ladder)

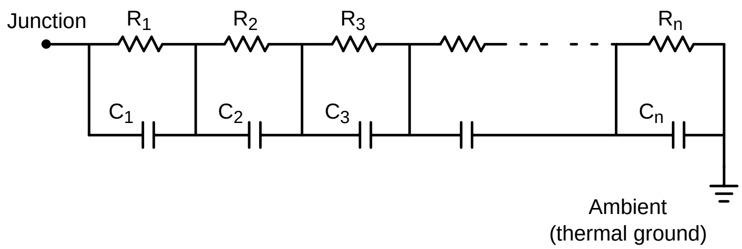

Figure 2. Non−Grounded Capacitor Thermal Ladder ("Foster" Ladder)

Non−Grounded−Capacitor Ladder

Contrast the grounded network of Figure 1, with the non−grounded−capacitor network of Figure 2. Figure 2 is a true "ladder" of resistors and capacitors, and is also known as a Foster ladder. Each rung is joined to the next rung (and only to the next rung) through both the resistor and the capacitor; only the capacitor of the ambient rung is directly connected to thermal ground.

Difficult though it may be to grasp at first, this network has no physical basis. In the real world, the thermal capacitance of a system is related to the change in temperature at each position in the system with respect to time − not with respect to temperatures elsewhere in the system (including adjacent elements). If we balk at this, it is because we understand intuitively that the temperature at one location must be coupled with the temperatures nearby (and indeed, this is precisely the information conveyed by the resistors in Figure 1). But consider fundamental, one−dimensional heat−transfer (as illustrated by Equation 1) which relates spatial derivatives of temperature (and conductivity) to the temporal derivative (and heat capacity).

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t} \qquad \text{(eq. 1)}$$

When we try to make the analogy with electrical capacitors, we get into trouble because the time derivative in the governing equation relating current to voltage contains a total time derivative of the voltage *difference* between the two ends of the capacitor; in other words, there is an implied spatial gradient in the derivative itself. Not so in the heat transfer equation, where the partial derivatives of space and time are explicit and distinct. (Indeed, in the basic electrical capacitor equation, there is no reference to resistance.) Why can't we refer to the temperature difference between the "ends" of a thermal capacitor? The short answer is this: in a thermal system, we only have access to one end of every thermal capacitor, namely the end we can measure − its temperature.

To put it another way, whether the "other end" of a thermal capacitor is at absolute zero, or at some hypothetical fixed reference temperature 7,320.12 degrees hotter than the surface of the sun, when we talk about thermal capacitance, we're really only interested in the change of energy storage in *time*, so we don't care about the "other end" of the capacitor (so long as it stays put). In a thermal transient analysis, though it matters a great deal how much energy is stored within each capacitor with respect to every other capacitor, at time zero, it makes no difference whether we refer them to each other or to the universal reference. Then *and only then*, the *differences* will be the same. Once time begins to march, however, we must continue to relate the stored energy to the original amounts (i.e. by monitoring the local change in temperature with respect to time), not to the also−time−varying neighboring amounts.

So back to the non−grounded−capacitor ladder. If it isn't real, why bother with it? Its justification is twofold: (1) it is nearly trivial to analyze in the mathematical sense, and (2) its overall transient response between the junction and ground can be tuned to match *exactly* the junction−to−ground response of a "real" network model such as Figure 1 (and more generally, *any* model − star, bridge, what−have−you).

As to the first point, we can appreciate the convenience most readily when we look into the methods used to analyze linear networks. The Laplace transform of the ladder of Figure 2 is simply the sum of terms whose amplitudes are the resistance values, and whose time constants are the RC products of each rung. Therefore, if for any *physical* system we have measured the transient response, there will exist some network of the form of Figure 2 which fits the data to arbitrary precision. All we need is to extract the amplitudes and time constants from the data, and we're done. Given the resulting network, we can then obviously use various techniques to solve it for any desired heat/temperature boundary conditions at the two ends.

As to the second point, though the junction node itself must have significance (since we've intentionally matched its response to real−world data), we must recognize immediately that any apparent resemblance of the intermediate nodes (between it and ground) to physical locations is purely coincidental. Clearly, if the overall response is simply the sum of a series of terms, each of which depends on only the properties of each rung independent of the others, we could rearrange the rungs into any order we please without changing the overall response. Try generating the Laplace transform of a Figure 1 network, and you'll quickly discover that the order matters immensely. It certainly violates our physical intuition to suppose that the transient response of a physical system doesn't care whether you put the heat in (or take it out) at a thermally "light" element or a thermally "heavy" element. There is clearly something "unreal" about a model that doesn't care.

So without belaboring it further, the easily analyzed network of Figure 2 has no physical significance, save that the junction node itself will have exactly the desired transient behavior. Freely transforming a system between its physically significant Figure 1 form and its Figure 2 equivalent is a considerably involved subject, and it is not necessary to address it here. Probably the single most important point is that the product of the R's and C's in the rungs of a grounded−capacitor network is *not* how you get the time constants; that only works for the non−grounded−capacitor network. By the same token, the R's in one network will not be the same as the R's in the other, nor will be the C's. About the only thing the two networks will have in common (at least if the Figure 1 network is a linear chain of resistors as shown), is that the overall sum of the R's will be the same, and the number of rungs will be the same.

It should be noted in passing that for more complex topologies of physically significant networks, two necessary (though insufficient) conditions for equivalence with a non−grounded RC ladder are these: (1) the two networks must have the same steady−state (or "DC") solution, hence they must exhibit a common overall system resistance from the junction node to ground; (2) in order to have identical transient behavior at the junction nodes, they must at least have identical sets of time constants, hence they must have the same total number of variable−temperature nodes. Since each variable−temperature node contributes one time constant to the system, for linear chains − that is to say, ladders − obviously the number of rungs must be the same.

How to Use the RC Ladder Networks

There are two basic alternatives in utilizing RC networks for calculating the transient response of a system. The strictly mathematical approach works well if (1) you are only interested in the junction behavior, and (2) you have the amplitudes and time constants of the transfer function of the system − in other words, you have obtained, by some method, the non−grounded RC network equivalent of the system. Given this scenario, the calculations are almost trivial, especially for the simplest situation of constant−power input at the junction node. Equation 2 puts this in mathematical terms, and Equation 3 expresses the same thing in a Microsoft Excel® formula using "array" syntax. In either case, of course, we're computing the junction temperature rise over ambient (assuming the system started uniformly at ambient).

$$\Delta T_j = Q * \sum R_i \left(1 - e^{-\frac{t}{\tau_j}}\right) \quad \text{(eq. 2)}$$

$$= \{\text{power} * \text{SUM(resists} * (1 - \text{EXP}(- \text{time/taus})))\} \quad \text{(eq. 3)}$$

where

"power" is a cell containing the constant power

"resists" is an array of resistors, such as A1:A7

"taus" is an array of associated time constants, such as B1:B7

"time" is a cell containing the time of interest

However, if you're interested in arbitrary time−varying power input, or are starting with the grounded−capacitor (i.e. physically significant) thermal network, the direct mathematical approach is much less convenient. Instead, a circuit simulator, such as SPICE, provides a flexible and straightforward method. You can use either the grounded or non−grounded capacitor models, but it is vitally important to enter the proper R's and C's of whichever version of the network you're working with, and obviously the appropriate network topology, into the simulator. If you've got the amplitudes and tau's of the real system (i.e. the results of a multiple−term, exponential fit to experimental data), you can easily generate the C's for a non−grounded circuit by dividing each tau by its associated R. Then, limited only by the features of your simulator, you can excite the network with any time−varying power inputs or other boundary conditions as desired.

One final reminder favoring the grounded−capacitor network over the non−grounded equivalent, is that because the nodes of the ladder bear some correlation with the physical package and system in which it resides, it may be possible to separate the package from its "environment," or to extract transient temperature behavior of unmeasured intermediate nodes from the model. The problem remains that the accuracy of any resulting sub−model can't be any better than the overall model from which it arose (for example, if the model did not comprehend known multiple, major heat−flow paths).

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